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Bayes estimators for reliability and hazard function of Rayleigh-Logarithmic (RL) distribution with application

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ABSTRACT

In this paper, we derived an estimators and parameters of Reliability and Hazard function of new mix distribution (Rayleigh-Logarithmic) with two parameters and increasing failure rate using Bayes Method with Square Error Loss function and Jeffery and conditional probability random variable of observation. The main objective of this study is to find the efficiency of the derived of Bayesian estimator compared to the to the Maximum Likelihood of this function using Simulation technique by Monte Carlo method under different Rayleigh-Logarithmic parameter and sample sizes. The consequences have shown that Bayes estimator has been more efficient than the maximum likelihood estimator in all sample sizes with application.

Keywords: Mix distribution Rayleigh- Logarithmic, reliability and hazard function, Maximum Likelihood estimator, the Bayesian estimator Jeffery formula

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1. Introduction

Rayleigh- Logarithmic (RL) distribution has been commonly used in reliability theory and survival analysis ,as its failure rate is a linear function of time. This distribution plays an important role in real life application including life testing and clinical studies of rate. Rayleigh- Logarithmic has two parameters: shape parameter(p), and the scale parameter (σ^2). It is constructed as a distribution of independent Rayleigh random variable s when the sample size K has a logarithmic distribution member of continuous probability distribution and considered as a model for failure time distribution this distribution is one of the earliest in probability theory, and it was introduced by Bugatekin [4] in 2017. In (2013) Al Mayali, and Al_Shaibani, have done A Comparison for some of the Estimators of Rayleigh Distribution with Simulation. In 2018, Al-obey has presented a Comparison of Bayes Estimators for the parameter of Rayleigh Distribution with Simulation. In 2019, Aldahlan, has applied a Classical and Bayesian Estimation for Toppleone Inverse Rayleigh Distribution. In 2020, Iqbal, Asma, and Lamyaa , have been derived Bayesian estimator Reliability function of Laplace distribution with two parameters (a,b) using Bayes Method with Square Error Loss function.

In this paper we derived estimator Reliability and Hazard function of new mix distribution (Rayleigh-Logarithmic) (RL) with two parameters and increasing failure rate using Bayes Method with Square Error Loss function and Jeffery and conditional probability random variable of observation $f(t_1, t_2, \dots, t_n / \rho, \sigma^2)$ based on a random sample of size (n) with distribution function F(t,θ) and the probability density function(t,θ). There are several steps to calculate the Bayes estimators of the (RL) distribution with two parameters (ρ, σ^2). Therefore, we must to know the prior distribution and posterior distribution as follows:

$$h(\rho, \sigma^2 / t_1, t_2, \dots \dots t_n) = \frac{f(t_1, t_2, \dots, t_n / \rho, \sigma^2) * g(\rho, \sigma^2)}{\int_0^1 \int_0^\infty f(t_1, t_2, \dots, t_n / \rho, \sigma^2) * g(\rho, \sigma^2) d\rho d\sigma^2}$$

We used , $L(\hat{\theta},\theta) = (\hat{\theta}-\theta)^2$, so the Bayes estimator $\widehat{R}(t)$ for Reliability $R(t)$ is

$\widehat{R}(t) = E[R(t)/t_1, t_2, \dots\dots, t_n)\,]$

And the Bayes estimator $\hat{h}(t)$ for hazard function $h(t)$ is:

$\hat{h}(t) = E[h(t)/t_1, t_2, \dots\dots, t_n]$

We used Monte Carlo Simulation and compare with the Maximum Likelihood Reliability function and Moment reliability of Rayleigh- Logarithmic(RL) distribution several values for the parameters and sizes ,simulation results have shown that Bayes estimator is the best method. Section 2 contains the derived of the Bayes estimators for Reliability and hazard of Rayleigh- Logarithmic (RL) distribution

Section 3 presents the theoretical part which it is contained the probability density function ,cumulative , moments of Rayleigh- Logarithmic distribution with two parameters as well , some properties of this distribution and then derived a Bayes estimator and the invariant Maximum Likelihood estimator and moment estimator for the Reliability function R(t) and hazard h(t). Section 4 provides a simulation study by using program R. Section 5 provides application of this study. Lastly, Section 6 presents the major conclusions of this study.

2. Material and methods

The probability density function for a Rayleigh- logarithmic (RL) distribution (p.d.f) with two parameters(ρ, σ^2) is :

$$f(t;\rho,\sigma^2) = -\frac{t}{\ln(1-\rho)\sigma^2} e^{-\frac{t^2}{2\sigma^2}} \rho \left(1 - e^{-\frac{t^2}{2\sigma^2}} \rho\right)^{-1} \quad \dots\dots(1)$$

Where,

$\sigma^2 > 0$ *scale prameter* ,

$0 < \rho < 1$ *shift prameter*

The cumulative distribution function (CDF) of Rayleigh- Logarithmic (RL) is :

$$F(t;\rho,\sigma^2) = 1 - \frac{\ln\left(1-\rho e^{\frac{-t^2}{2\sigma^2}}\right)}{\ln(1-\rho)} \quad \dots\dots\dots\dots\dots(2)$$

Therefore , the reliability function is:

$R(t;\rho,\sigma^2) = 1 - F(t;\rho,\sigma^2)$

$$R(t;\rho,\sigma^2) = \frac{\ln\left(1-\rho e^{-\frac{t^2}{2\sigma^2}}\right)}{\ln(1-\rho)} \quad \dots\dots\dots\dots(3)$$

With hazard, the function is written as :

$$h(t;\rho,\sigma^2) = \frac{f(t;\rho,\sigma^2)}{R(t;\rho,\sigma^2)} = -\frac{t e^{-\frac{t^2}{2\sigma^2}} p\left(1-pe^{-\frac{t^2}{2\sigma^2}}\right)^{-1}}{\sigma^2 \ln\left(1-e^{-\frac{t^2}{2\sigma^2}}\rho\right)} \quad \dots\dots\dots(4)$$

The likelihood function will for sample of size n from (ρ, σ^2) reliability function for Rayleigh- Logarithmic Distribution is feasibly derived by:

$$L(t;\rho,\sigma^2) = \prod_{i=1}^{n} f(t_i;\rho,\sigma^2)$$
$$= (-1)^n \frac{\prod_{i=1}^{n} t_i}{(\sigma^2)^n[\ln(1-\rho)]^n} e^{-\frac{\sum_{i=1}^{n} ti^2}{2\sigma^2}} \rho^n \prod_{i=1}^{n} (1-\rho e^{-\frac{t_i^2}{2\sigma^2}})^{-1}$$

$$\text{Ln L}(\text{ t};\ \rho,\sigma^2) = \ln(-1)n + \sum_{i=1}^{n} \ln t_i - n\ln\sigma^2 - n\ln\left[\ln(1-\rho)\right] - \sum_{i=1}^{n} \frac{t_i^2}{2\sigma^2} + n\ln\rho - \sum_{i=1}^{n} \ln(1-\rho e^{\frac{-t_i^2}{2\sigma^2}})^{-1}$$

$$\frac{\partial \ln l}{\partial \rho} = \frac{n}{(1-\hat{\rho})\ln(1-\hat{\rho})} + \frac{n}{\hat{\rho}} + \sum_{i=1}^{n} \frac{e^{-\frac{t_{i^2}}{2\hat{\sigma}^2}}}{1-\hat{\rho}e^{-\frac{t_{i^2}}{2\hat{\sigma}^2}}} = 0$$

And in the same way concerning (σ^2), we will get the following :

$$\frac{\partial \ln l}{\partial \sigma^2} = -\frac{n}{\sigma^2} * (1) - \sum_{i=1}^{n} -\frac{t_i^2}{2} - (\sigma^2)^{-2} + \sum_{i=1}^{n} \frac{-pe^{-\frac{t_{i^2}}{2\sigma^2}} - \frac{t_{i^2}}{2} * -(\sigma^2)^{-2}}{1-pe^{-\frac{t_{i^2}}{\sigma^2}}} = 0$$

By solving these equations by using the simulation method (f.solve), we get the value of the estimators $\hat{\sigma}_{mle}$ and $\hat{\rho}_{mle}$

Therefore, the property of invariant maximum likelihood estimator of the hazard function of (RL):

$$\hat{h}_{mle}(t) = -\frac{te^{\frac{-t^2}{2\hat{\sigma}^2}}\hat{p}\left(1-\hat{p}e^{\frac{-t^2}{2\hat{\sigma}^2}}\right)^{-1}}{\hat{\sigma}^2\ln\left(1-e^{\frac{-t^2}{2\hat{\sigma}^2}}\hat{p}\right)} \qquad \ldots\ldots(5)$$

And maximum likelihood estimator of the reliability function of(RL) will be:

$$\hat{R}_{mle}(t,p,\sigma^2) = \frac{\ln\left(1-\hat{p}e^{\frac{-t^2}{2\hat{\sigma}^2}}\right)}{\ln(1-\hat{p})} \qquad \ldots\ldots(6)$$

Our suggested Bayes Estimator was derived as following:

In this section, we estimate the parameters (ρ,σ^2) of Rayleigh- logarithmic distribution by Bayesian method by using square error loss function (SE).

Let (t1, t2,…., tn) be a random sample of size (n) with distribution function F(t,θ) and the probability density function(t,θ), there are several steps to calculate the bayes estimators of the (RL) distribution with two parameters (p,σ^2).

So we must to know the prior distribution and posterior distribution as follows:

Posterior distribution= $\frac{prior\ distribution*likelihood\ function}{marginal\ distribution}$

According to the general definition, if we consider that:

$$g_1(\theta)\alpha\frac{1}{\theta}; \qquad \theta > 0\,, \qquad \theta = \sigma^2$$

Not that (p) follows uniformed distribution and (σ^2) Uniform logarithmic distribution.

The conditional density function is given by:

$$g_2(p) = 1, \qquad 0 < p < 1$$

Then:$g(p,\sigma^2)\ \alpha\ g_1(\sigma^2) * g_2(p)$

$$g(p,\sigma^2) = \frac{k}{\sigma^2} \qquad \text{where k is constant}$$

The probability density function for the two parameters (p,σ^2) is:

$$h(\rho,\sigma^2/\,t_1,t_2,\ldots\ldots t_n) = \frac{f(t_1,t_2,\ldots.,t_n\,/p,\sigma^2) * g(\rho,\sigma^2)}{\int_0^1\int_0^\infty f(t_1,t_2,\ldots\ldots,t_n/p,\sigma^2) * g(\rho,\sigma^2)dpd\sigma^2}$$

$$= \frac{-k \frac{\prod_{i=1}^{n} t_i}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-\sum_{i=1}^{n} \frac{t_{i2}}{2\sigma^2}} p^n \prod_{i=1}^{n} \left(1 - pe^{\frac{-t_{i2}}{2\sigma^2}}\right)^{-1}}{-k \int_0^1 \int_0^\infty \frac{\prod_{i=1}^{n} t_i}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-\sum_{i=1}^{n} \frac{t_{i2}}{2\sigma^2}} \; p^n \prod_{i=1}^{n} \left(1 - pe^{\frac{-t_i^2}{2\sigma^2}}\right)^{-1} dpd\sigma^2}$$

$$= \frac{\frac{p^n \prod_{i=1}^{n} t_i}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-\sum_{i=1}^{n} (\frac{t_i^2}{2\sigma^2} + \ln\left(1 - pe^{\frac{-t_i^2}{2\sigma^2}}\right))}}{\int_0^1 \int_0^\infty \frac{p^n \prod_{i=1}^{n} t_i}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-\sum_{i=1}^{n} (\frac{t_i^2}{2\sigma^2} + \ln\left(1 - pe^{\frac{-t_i^2}{2\sigma^2}}\right))} dpd\sigma^2}$$

To obtain Bayes' estimator, we minimize the posterior expected loss as follows:

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$$

After simplified steps, we get Bayes estimator for (p) denoted by ($\hat{p}_{B.E}$), for above prior as follows:

$$h_1(p/t_1, t_2, \dots . t_n) = \frac{\int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]} e^{-w} \, d\sigma^2}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]} e^{-w} dpd\sigma^2}$$

$$w = \sum_{i=1}^{n} \left[\frac{t_i^2}{2\sigma^2} + \ln\left(1 - pe^{-\frac{t_i^2}{2\sigma^2}}\right)\right]$$

$$\hat{p}_{B.E} = E(p/t_1, t_2, \dots \dots, t_n) = \frac{\int_0^1 \int_0^\infty \frac{p^{n+1}}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} d\sigma^2}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} dpd\sigma^2} \quad \dots \dots \qquad (7)$$

Also to the parameter(σ^2):

$$h_2(\sigma^2/t_1, t_2, \dots \dots, t_n) = \frac{\int_0^1 \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} \, dp}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} dpd\sigma^2}$$

$$\hat{\sigma}^2{}_{B.E} = E(\sigma^2/t_1, t_2, \dots \dots, t_n) = \frac{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^n [\ln(1-p)]^n} e^{-w} dp \, d\sigma^2}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} dp \, d\sigma^2} \quad \dots \dots . . (8)$$

In addition, we depended on the values of the estimates using the squared loss function, the estimator of the hazard function is:

$$\hat{h}(t) = E[h(t)/t_1, t_2, \dots . . , t_n]$$

$$\hat{h}_{B.E}(t) = \frac{-\int_0^1 \int_0^\infty \frac{tp^{n+1}}{(\sigma^2)^{n+2}[\ln(1-p)]^n} * \frac{e^{-\frac{t^2}{2\sigma^2}}}{\ln\left(1 - pe^{-\frac{t^2}{2\sigma^2}}\right)} * e^{-w} dp \, d\sigma^2}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^n} e^{-w} dpd\sigma^2} \quad \dots \dots \dots . (9)$$

And for the reliability Bayes estimator:

$$\hat{R}(t) = E[R(t)/t_1, t_2, \ldots\ldots, t_n)\ ]$$

$$\hat{R}_{B.E}(t) = \frac{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^{n+1}} \ln\left(1-pe^{-\frac{t^2}{2\sigma^2}}\right) e^{-w} dp\, d\sigma^2}{\int_0^1 \int_0^\infty \frac{p^n}{(\sigma^2)^{n+1}[\ln(1-p)]^{n}} e^{-w} dp d\sigma^2} \quad \ldots\ldots\ldots(10)$$

3. Simulation study

In simulation we adopted r=1000 where r is the replications .Arbitrary values of parameters shift (p = 0.2, 0.4) and scale (($\boldsymbol{\sigma^2}$= 4 ,5) parameters respectively .
Arbitrary values of sample sizes as (n=30, 70, 130) from Rayleigh- Logarithmic distribution were selected by using R simulator version .
Rayleigh- Logarithmic distribution has generated on:

$$t_i = [\ln \left[\frac{1-(1-p)^{(1-u)}}{p}\right]^{2\sigma^2}]^{\frac{1}{2}}$$

Then, the values of hazard and reliability of the maximum likelihood have computed according to equations (5) and (6) , the values of hazard and reliability of the Bayes have computed according to equations (9) and (10) .
Finally, we computed the efficiency of the estimators using Integrative Mean Squares Error (IMSE).

$$\mathrm{IMSE}[\hat{h}(t)] = \frac{1}{r}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}[\hat{h}_i(t_j) - h(t_j)]^2\right\}$$

$$\mathrm{IMSE}[\hat{R}(t)] = \frac{1}{r}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}[\hat{R}_i(t_j) - R(t_j)]^2\right\}$$

$\boldsymbol{h(t_j)}$ and $\boldsymbol{R(t_j)}$ are the real values of the hazard and reliability respectively .
$\boldsymbol{\hat{h}_i(t_j)}$ and $\boldsymbol{\hat{R}_i(t_j)}$ are the estimators of the hazard and reliability respectively according to the method

4. Results and discussion

The results of the estimator and different samples sizes are depicted in details in the following tables:

Table 1. The values of IMSE for estimators of R(t) with different sample sizes

$\boldsymbol{\sigma^2 = 4}$, $\boldsymbol{p = 0.2}$			
Method n	**MLE**	**Bayes**	**Best**
30	0.00309215	0.00135339	Bayes
70	0.00124351	0.00115565	Bayes
100	0.00095226	0.00093126	Bayes
130	0.00091130	0.00087938	Bayes

From Table 1, when($\boldsymbol{\sigma^2 = 4}$, $\boldsymbol{p = 0.2}$) with different samples sizes (n=30, 70, 100, 130), the best method of estimator R(t) is Bayes method because of smallest value of IMSE.

Table 2. The values of IMSE for estimators of H(t) with different sample sizes

$\sigma^2 = 4$, $p = 0.2$			
Method N	**MLE**	**Bayes**	**Best**
30	0.02308790	0.01222825	Bayes
70	0.01009045	0.00927833	Bayes
100	0.00821054	0.00698224	Bayes
130	0.00723087	0.00621171	Bayes

From Table 2, when($\boldsymbol{\sigma^2 = 4}$, $\boldsymbol{p = 0.2}$) and (n=30,70,100,130), the best method of estimator H(t) is Bayes method for all sample sizes.

Table 3. The values of IMSE for estimators of R(t) with different sample sizes

$\sigma^2 = 5$, $p = 0.4$			
Method N	**MLE**	**Bayes**	**Best**
30	0.00303723	0.00194354	**Bayes**
70	0.00189047	0.00149682	**Bayes**
100	0.00121485	0.00117582	**Bayes**
130	0.00258183	0.00114166	**Bayes**

Based on Table 3, when($\boldsymbol{\sigma^2 = 5}$, $\boldsymbol{p = 0.4}$) and (n=30,70,100,130), the best method of estimator R(t) is Bayes method because of smallest value IMSE

Table 4. The values of IMSE for estimators of H(t) with different sample sizes

$\sigma^2 = 5$, $p = 0.4$			
Method N	**MLE**	**Bayes**	**Best**
30	0.01874175	0.015633300	**Bayes**
70	0.00966182	0.00582258	**Bayes**
100	0.00941306	0.00661884	**Bayes**
130	0.00904502	0.00603021	**Bayes**

Based on Table 4, when ($\boldsymbol{\sigma^2 = 5}$, $\boldsymbol{p = 0.4}$) and (n = 30, 70, 100, 130), the finest method of estimator R(t) is Bayes method because of smallest value of IMSE

5. Application with real data

Real data of 100 machines for the maintenance of Watering and sizzle projects in Baghdad has been collected with failure time of the machines in months(t_i). We use The Bayes estimators of R(t) and H(t) of Rayleigh-logarithmic(RL) in application with the real data of 100 machines because it is the best with smallest IMES in the table below :

Table 5. The value of $\hat{R}(t)$ and $\hat{H}(t)$of Bayes estimator for the real data failure time of the machines in months(ti)

n	t_i	$\hat{R}(t)$_Bayes	$\hat{H}(t)$_Bayes	t_i	$\hat{R}(t)$_Bayes	$\hat{H}(t)$_Bayes
100	0.3	0.986330	0.091616	2.7	0.359431	0.713875
	0.3	0.986330	0.091616	2.8	0.334294	0.736146
	0.4	0.975856	0.121855	2.8	0.334294	0.736146
	0.5	0.962591	0.151845	2.8	0.334294	0.736146
	0.6	0.946676	0.181536	2.8	0.334294	0.736146
	0.8	0.907590	0.239844	2.9	0.310224	0.758331
	0.8	0.907590	0.239844	2.7	0.359431	0.713875
	0.8	0.907590	0.239844	2.8	0.334294	0.736146
	0.8	0.907590	0.239844	2.8	0.334294	0.736146
	0.9	0.884814	0.268394	2.8	0.334294	0.736146
	0.9	0.884814	0.268394	3	0.287251	0.780455
	0.9	0.884814	0.268394	3	0.287251	0.780455
	1	0.860169	0.296507	3.1	0.265391	0.802539

n	t_i	$\hat{R}(t)$_Bayes	$\hat{H}(t)$_Bayes	t_i	$\hat{R}(t)$_Bayes	$\hat{H}(t)$_Bayes
	1	0.860169	0.296507	3.1	0.265391	0.802539
	1	0.860169	0.296507	3.2	0.244655	0.824601
	1	0.860169	0.296507	3.2	0.244655	0.824601
	1.1	0.833881	0.324169	3.2	0.244655	0.824601
	1.2	0.806183	0.351372	3.3	0.225042	0.846661
	1.2	0.806183	0.351372	3.3	0.225042	0.846661
	1.2	0.806183	0.351372	3.3	0.225042	0.846661
	1.3	0.777305	0.378113	3.4	0.206545	0.868733
	1.3	0.777305	0.378113	3.4	0.206545	0.868733
	1.4	0.747476	0.404399	3.4	0.206545	0.868733
	1.4	0.747476	0.404399	3.5	0.189150	0.890832
	1.4	0.747476	0.404399	3.6	0.172837	0.912972
	1.5	0.716922	0.430239	3.7	0.157581	0.935162
	1.5	0.716922	0.430239	3.7	0.157581	0.935162
	1.5	0.716922	0.430239	3.7	0.157581	0.935162
	1.6	0.685857	0.455649	3.8	0.143354	0.957414
	1.6	0.685857	0.455649	3.9	0.130120	0.979736
	1.7	0.654487	0.480649	4	0.117844	0.979806
	1.7	0.654487	0.480649	4.1	0.106488	0.979880
	1.7	0.654487	0.480649	4.1	0.106488	0.979901
	1.7	0.654487	0.480649	4.1	0.106488	0.979932
	1.7	0.654487	0.480649	4.3	0.086366	0.979970
	1.8	0.623004	0.505261	4.5	0.069413	0.980214
	1.8	0.623004	0.505261	4.5	0.069413	0.983210
	1.9	0.591588	0.529510	4.7	0.055279	0.985021
	1.9	0.591588	0.529510	4.8	0.049161	0.987561
	2	0.560406	0.553424	5.1	0.034100	0.989201
	2	0.560406	0.553424	5.1	0.034100	0.989902
	2	0.560406	0.553424	5.3	0.026410	0.990235
	2.1	0.529608	0.577032	5.7	0.015398	0.992350
	2.1	0.529608	0.577032	5.9	0.011591	0.995320
	2.2	0.499329	0.600362	6.3	0.006382	0.997202
	2.2	0.499329	0.600362			
	2.3	0.469690	0.623444			
	2.4	0.440796	0.646309			
	2.4	0.440796	0.646309			
	2.4	0.440796	0.646309			
	2.4	0.440796	0.646309			
	2.4	0.440796	0.646309			
	2.5	0.412740	0.668983			
	2.5	0.412740	0.668983			
	2.5	0.412740	0.668983			
	2.5	0.412740	0.668983			
	2.6	0.385597	0.691497			
	2.6	0.385597	0.691497			
	2.7	0.359431	0.713875			

6. Conclusions

The Bayes estimators of R(t) and H(t) of Rayleigh-Logarithmic (RL) are most efficient than MLE because of the smallest value of IMSE with all sample size. The IMSE has been decreasing with big sample sizes.

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